

RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

FIRST YEAR [BATCH 2017-20]

B.A./B.Sc. SECOND SEMESTER (January – June) 2018

Mid-Semester Examination, March 2018

Date : 17/03/2018

MATHEMATICS (General)

Time : 11 am – 12 noon

Paper : II

Full Marks : 25

[Use a separate Answer Book for each group]

Group – A

[10 marks]

Unit - I

1. Answer **any one** question :

[1×5]

- a) The equation $3x^2 + 2xy + 3y^2 - 18x - 22y + 50 = 0$ is reduced to $4x^2 + 2y^2 = 1$ when referred to rectangular axes through the point (2, 3). Find the inclination of the latter axes to the former.
- b) If $ax + by$ transforms $a'x' + b'y'$ under rotation of axes then show that $a^2 + b^2 = a'^2 + b'^2$.

Unit - II

2. Answer **any one** question :

[1×5]

- a) i) Prove that for any vectors $\vec{a}, \vec{b}, \vec{c}$ we have $\vec{a} \times (\vec{b} \times \vec{c}) + \vec{b} \times (\vec{c} \times \vec{a}) + \vec{c} \times (\vec{a} \times \vec{b}) = \vec{0}$. [2]
- ii) Calculate the shortest distance between two straight lines AB & CD where $A(-1, 2, -3)$, $B(-16, 6, 4)$, $C(1, -1, 3)$ & $D(4, 9, 7)$. [3]
- b) i) Find the equation of the plane which passes through two given points $A(\vec{a})$ and $B(\vec{b})$ and which is parallel to a given vector $\vec{c}$. [2.5]
- ii) Find the value of the constant 'd', such that the vectors $(2\hat{i} - \hat{j} + \hat{k})$, $(\hat{i} + 2\hat{j} - 3\hat{k})$ and $(3\hat{i} + d\hat{j} + 5\hat{k})$ are coplanar. [2.5]

Group – B

[15 marks]

Unit - III

3. Answer **any three** questions :

[3×4]

- a) Prove that $\left\{ \frac{3n^2 + 2n + 1}{7n^2 + 8n + 9} \right\}$ is convergent.
- b) Every convergent sequence is bounded. Does the converse holds good? Justify your answer.
- c) Prove that $\left\{ \frac{1}{n^2} \right\}$ is a Cauchy sequence.
- d) Show that $\{x_n\}$ is convergent, where $x_n = 1 + \frac{1}{2!} + \frac{1}{3!} + \dots + \frac{1}{n!}$.
- e) Prove, using Cauchy's general principle of convergence that the series $\sum_{n=1}^{\infty} \frac{1}{n}$ is not convergent.
- f) Test the convergence if the series $1 + \frac{1}{2} \cdot \frac{1}{3} + \frac{1 \cdot 3}{2 \cdot 4} \cdot \frac{1}{5} + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} \cdot \frac{1}{7} + \dots$.

Unit - IV

4. Answer **any one** question :

[1×3]

a) Solve : $\frac{dy}{dx} = \frac{x+y+4}{x-y-6}$.

b) Solve : $(x^2y - 2xy^2)dx - (x^3 - 3x^2y)dy = 0$.

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